Book Review

Dynamic Geometry on Time Scales

This book provides a comprehensive introduction to differential geometric curves and surfaces on time scales as well as higher dimensional objects. It is a relatively new theory, originating in the dissertation of Stefan Hilger (1988) under the supervision of Bernard Aulbach. In principle, it unifies the two approaches to dynamic modeling, i.e., difference and differential equations. The author uses the tools of time scale calculus, establishing the general theories of curves and surfaces in $\mathbb{R}^n$; supporting them with numerous examples and advanced practical problems, and then providing the counterpart of the classical differential geometric results.

The content of the book is organized in seven chapters: Curves in $\mathbb{R}^n$, General Theory of Surfaces, First Fundamental Forms, The Second Fundamental Forms, The Fundamental Equations of a Surface, Minimal Surfaces, The Delta Nature Connection.

In the first chapter, Curves in $\mathbb{R}^n$; the author essentially aims at presenting the counterpart on time scales to Fundamental Theorem for a space curve, a uniqueness and existence theorem. For this purpose, the whole necessary background is built up.

In the second chapter, General Theory of Surfaces, surfaces are studied with emphasis on parametric and implicit repsentations. Subsequently, the notions of tangent plane, normal vector and orientability are introduced.

The third chapter, First Fundamental Forms, introduces the metric concepts on a parameterized surface such as length, angle and area.

The fourth chapter, The Second Fundamental Forms, deals with two classical theorems: the Meusnier and the Joachimstahl theorems. In addition, the curvature invariants of a surface are examined, such as the normal, the Gaussian and the mean curvature. Their properties are well studied.

In the fifth chapter, The Fundamental Equations of a Surface, a uniqueness and existence theorem for a surface is established by means of the Gauss and Godazzi-Mainardi equations. Several concepts are also introduced for a curve on a surface to be a geodesic.

The sixth chapter, Minimal Surfaces, considers a classical variational problem, i.e., the problem of finding a surface locally minimizing its area. Several examples of minimal surfaces are also given.

In the last chapter, the author introduces new types of derivatives and examines their properties: the directional derivative, the covariant differentiation, the delta nature connection and the exterior differentiation.

The content of the reviewed book is presented in an accessible manner. Many new ideas and results are presented and well-explained. Most theorems and corollaries are given along with elegant proofs. The author's arguments and computations are based on knowledge of differential geometry and time scale calculus. Therefore, the reviewer feels that the book will be well beyond the scope of a typical undergraduate and graduate course.

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